Buletin Teknol. Tanaman, Bil.12, Tambahan.1 (2015): 210-211

An Enhanced Optimization of Hooke-Jeeves Approach

Seyyed Mohammad Tabatabaei Mehrizi, Ali Sorourkhah

Abstract
This article aimed at explaining Hooke-Jeeves method, which is addressed as an ineffective approach to achieve optimum solutions to some problems in the following. At the end, an enhanced approach is suggested for the convergence and achievement of optimal results.
Key words: Hooke-Jeeves method, non-differentiable multivariate functions, infinite functions, optimization

Introduction
Optimization techniques are basically divided into two groups of direct search with and without the use of derivatives. The mentioned approaches are called analytical methods when differential calculus methods are used for the optimization of functions. Optimization techniques without the use of derivatives are more useful in practice since on the one hand, objective functions cannot be analytically determined and it is not possible for them to be derived and on the other hand, it is not simple to algebraically determine the static points. Therefore, the above-mentioned methods are somewhat acceptable to estimate or approximate the locations of one or more optimal points. To date, a variety of methods have been presented for the minimization of non-differentiable n-variable and infinite functions, from among which the method of rotating coordinates, Rosenbrock's method, the simplex method, etc. could be mentioned. A relatively simple and effective method in this field is Hooke-Jeeves method (with continuous and discontinuous processes). The procedure in this method is such that (n-1) variables are kept at a constant level and the search is applied to the nth variable so as to achieve the maximum or minimum of the desired function. Then, the nth variable is stabilized within the value obtained and another variable, e.g. n-1th variable, is selected. This is continued as far as no improvement in the value of the objective function is achieved by any changes in any variables [2].

Theoretical fundamentals and research background
Hooke-Jeeves algorithm, which is a sequential optimization procedure, was first presented in 1961. There are two types of heuristic and pattern searches through its steps of iterative calculations [3]. In the heuristic search, local behavior of the objective function is determined and in pattern search, orientation of the objective function is designated [4]. In this algorithm, the starting or initial point (X_0), possible helpful directions (d_j), maximum acceptable distance ($\boldsymbol{\varepsilon}$), and the first step-length of acceleration ($\Delta \mathbf{x}$) and its coefficient ($\propto$) are first determined. In Hooke-Jeeves discontinuous optimization method, each step goes to the next according to movement in the desired direction (successful movement) or movement in the opposite of the desired direction (unacceptable point of the function) and the search process continues until an optimal point (or close to it) is achieved. In case of not arriving at a more suitable result, Δ or $\propto$has to be changed. In certain conditions such as improper selection of the starting point or specific type of objective function, this process can lead to the prolongation of investigation. Through providing an example in this article, it has been shown that by assuming an acceptable solution to the unacceptable result of the pattern search and repeating it with a heuristic search, an optimal answer can be obtained faster and without having to change Δ or $\propto$ [1].
The main steps of this method would be as follows:
A) Heuristic search
1. If we have $f(y_j + \mathbf{\Delta} x\, d_j) < f(y_j)$, then, the movement will be successful and we will go to the second step. Otherwise, the minimizing movement will be unsuccessful and the new point will not be accepted. In this case, the new value of variable y is determined in a negative direction ($-\Delta \boldsymbol{y}$):
$Y_{j+1} = y_j - \Delta \boldsymbol{x}\, \boldsymbol{d}_j$
If we have $f(y_j - \mathbf{\Delta} x\, d_j) < f(y_j)$, then, the movement will be successful and we will go to the second step. Otherwise, y value is kept constant and we will go to the second step:
$Y_{j+1} = y_j$
2. If $j< \boldsymbol{n}$, j+1 will be replaced by j and the first step will be repeated. However, if relation $f(y_{n+1}) < f(x_k)$ is set, we should go to the 3rd step and if relation $F(y_{n+1}) \geq f(x_k)$ is established, we should go to the 4th step.
B) Pattern search
3. Acceleration step with an assumption:
$X_{k+1} = y_{n+1}$, $Xp_{k+1} = y_1 = X_{k+1} = X_{k+1} + \propto (X_{k+1} - X_k)$; $(X_{k+1} - X_k)$ Acceleration rate
Considering j=1, k+1 is replaced by K and we will go to the 1st step.
4. If $\Delta \boldsymbol{x} \leq \boldsymbol{\varepsilon}$, optimization process will be discontinued and the problem solving procedure stops. Otherwise, $\Delta \boldsymbol{x}$ will be divided by 2 and the process of problem solving will continue. In case of not reaching an optimal solution, it is better to consider different values for $\Delta \boldsymbol{x}$ to be drawn along various directions.
Inefficiency of Hooke-Jeeves discontinuous method and presentation of the proposed approach to accelerate finding an optimal point
It can be observed that in case of not arriving at a better result in the pattern search, $\Delta \boldsymbol{x}$ and $\propto$ could be redirected and the searching process be continued. Now, it can be said that by assuming as acceptable the unacceptable result of the pattern

search and continuing the method with the heuristic search without changing $\Delta\boldsymbol{x}$ or $\propto$, an optimal answer can be achieved more quickly. This is shown below with an example.

Case study

Minimization of the following multivariate function is assumed:

$$\boldsymbol{f(x) = \left(x_1^3 - x_2\right)^2 + 2(x_2 - x_1)^4}$$

$$\boldsymbol{x_0} = \binom{-3}{-3}\ ,\ \Delta\boldsymbol{x} = \binom{1}{1}$$

$$f(x_0 = -3\,, -3) = 576$$

1st iteration:

1. Heuristic search

$$\begin{cases} x_1 = -3 + 1 = -2 \\ x_2 = -3 \quad \text{Constant} \\ f(-2,-3) = 27 < 576 \quad \text{Successful movement} \end{cases}$$

$$\begin{cases} x_1 = -2 \quad \text{Constant} \\ x_2 = -3 + 1 = -2 \\ f(-2,-2) = 36 > 27 \quad f(x)\ \text{Unaccepted} \end{cases}$$

$$\begin{cases} x_1 = -2 \quad \text{Constant} \\ x_2 = -3 - 1 = -4 \\ f(-2,-4) = 48 > 27 \quad f(x)\ \text{Unaccepted} \end{cases}$$

2. Pattern search

$$x_{p_1} = \binom{-2}{-3} + \left[\binom{-2}{-3} - \binom{-3}{-3}\right] = \binom{-1}{-3}$$

$$f(-1,-3) = 36 > 27 \quad f(x)\ \text{Unaccepted}$$

Assumption: we assume the unacceptable point $(-1,-3)$ as acceptable and then, we will go to the 2nd iteration and do the heuristic search.

2nd iteration:

1. Heuristic search

$$\begin{cases} x_1 = -1 + 1 = 0 \\ x_2 = -3 \quad \text{Constant} \\ f(0,-3) = 166 > 27 \quad f(x)\ \text{Unaccepted} \end{cases}$$

$$\begin{cases} x_1 = -1 \quad \text{Constant} \\ x_2 = -3 + 1 = -2 \\ f(-1,-2) = 3 < 27 \quad \text{Successful movement} \end{cases}$$

2. ***Pattern search***

$$x_{p_2} = \binom{-1}{-2} + \left[\binom{-1}{-2} - \binom{-1}{-3}\right] = \binom{-1}{-1}$$

$$f(-1,-1) = 0 < 3 \quad \text{Successful movement}$$

Optimal answer:

$$x_1^* = -1 \quad , \quad x_2^* = -1 \quad , \quad f(x) = 0$$

Suggestions

Hooke-Jeeves sequential discontinuous search method is a relatively simple method from among those aimed at achieving a minimum objective function within infinite multivariate functions without any limitations. Although this method has relatively good computational performance in reaching the target, its efficiency and speed can be enhanced to obtain a minimum point. One way to achieve this purpose as shown in the example above would be adding a simple assumption to the pattern and continuing the problem solving process, a case in which no changes and repeated calculations will be needed. Considering this assumption can prevent iterative calculations and make possible the achievement of an optimum solution.

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Seyyed Mohammad Tabatabaei Mehrizi, Ph.D Student of Industrial Management, Operation Research, Semnan University, Semnan, Iran
Email:seyyedmohammadtabatabaei@semnan.ac.ir
Ali Sorourkhah, Ph.D Student of Industrial Management, Operation Research, Semnan University, Semnan, Iran
Email:ali.sorourkhah@semnan.ac.ir