The Effect of Temperature on the Natural Frequency

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Abstract
In the majority of systems that have a stimulus part, in addition to the limitations of geometry and structure, another factor affecting on system design, this factor is the system vibrations, which, if uncontrolled, can be very damaging. On the other hand, some systems like brakes, combustion engines or even building structures, used in different temperature ranges and this temperature changes due to the impact on the structure of matter also changes the vibration characteristics of the system. Explicitly, it can be said that the natural frequency is the most important factor on vibration of a system and this factor depends on the geometry and material system. If this property be equal with frequency of external force that affecting the system, it can cause of serious damage to the system. Therefore, in this project, vibration properties of a beam at different temperatures are checked. Try to make a specific pattern for these changes in the operating temperature range. That after the experiment concluded that the natural frequency of the system with damper not much changes with temperature changing.

Keywords: natural frequency, the relationship between temperature and vibrations, system with and without damping, effect of temperature.

Introduction
Most human activities are dealing with vibrations. For example, we hear, because the eardrums vibrate, we see, because light waves oscillate. We breathing with the oscillation of lungs, we walking with vibration motion of our hands and feet, and we speak the oscillating motion tongue. Most of machines also have oscillation problems caused by their unbalance motors, for example, unbalanced diesel engines make the noise in the surrounding by creating powerful waves. Some locomotive unbalance wheel at high speeds may be rise up more than one centimeter of the track. Vibrations caused the failure of the turbine parts. Engineers still failed to prevent from the breaking turbine blades and wheels, which they are vibrate. Basically, the structure that designed for supports the heavy centrifuge machines (such as motors and turbines) or reciprocating machines are will oscillate. In all these cases, moving parts may be break caused by repetitive stress and fatigue [1]. It can be said most important concept is the vibration is the natural frequency that influenced most of the mechanical design especially the design of systems that are required to eliminate vibrations. It should be noted that the natural frequency only depend on physical state of the system. For each system can be considered one or more natural frequencies - depending on the degree of freedom - that depend on the system physical properties. On the other hand, always can be considered a frequency for foreign power and this parameter is known on vibration analysis. Problems caused by vibrations when being critical that the frequency of external force be equal to the system’s natural frequency and resonance occurs, in which case the vibration amplitude reaches its maximum. This is undesirable, as shown in Figure 1 and it destroyed a bridge. To fix the adverse effects, system should be designed so that the Natural frequency(s) have an acceptable difference with frequency of the external force.

If we take a closer look at the factors affecting the natural frequency can be divided into two general categories: Permanent factors such as size, type, density and grain structure, which is constant considered. Temporary factors, such as the life of the system and the working temperature always are changing. Here the temperature effect on the natural frequency - because of its high importance in changing the system properties - is discussed. Including more important ones can be aircraft wing vibrations at different altitudes - due to changes in temperature with height - vibration of the front of the space shuttle or aircraft at high speeds - due to sensible heat by friction - vibrations of buildings including bridges, brake vibrations - such as automobile brake - or any other system that is used at different temperatures.
Generally, the temperature effect on the vibrational properties of the structure is in three ways: first, the change in modulus of elasticity, second, changes in structural damping and Third, effect of temperature on the size. According to a study conducted by the University of Hong Kong [2] to investigate the effect of temperature gradient on the vibrations of a bridge, by using accelerometer devices installing them in certain locations of tested beams and then using linear regression and finally to achieve desired, development of these tests showed that the temperature rise will reduce the natural frequency. However, these measurements were performed on a normal day temperature changes and changes in temperature was from about 23 to 38 ° C. In another study on the effects of temperature on a plate vibration is done, based on mathematical modeling reached to the similar results with above paragraph, the natural frequency decreases with increasing temperature [3]. In the journal published by R-T-A suggested that effect of temperature on the natural frequency reviewing by use of ultrasound devices (device sonic TC) and at the end it was noted that for design should be considered about 3% tolerance, to effect of temperature changes, on natural frequency. It is worth mentioning at this research were not mentioned anything about the nature and high-temperature and temperature that discussed was 12 to 35 ° C [4]. Having recognized the importance and influence of temperature on its natural frequency, it can be stated that it is important and necessary to evaluate the effect of temperature in the systems that is work in large temperature range. To study the effect of temperature changes on the natural frequency, as examples a beam vibration has been analyzed. To be more specific vibrations, was used a steel belt as a beam and by putting it into a workshop clamp, beam was simulated. And continue with specific mechanism to put the beam at the insulated chamber. After making the beam conditions and insulation, the tests are beginning at the two modes, one, static experiment - by loading and measured deflection - and tow, dynamic experiment - by measuring the parameters of forced vibrations - the effects of changing temperature on the natural frequency is investigated. We can say that the use of tests to evaluate the issue is that in theory never can accurately involve effect of factors such as structural damping and volume expansion’s on the calculations and therefore in most of the research that was listed in the previous section has been used from the test to obtain the results.

EXPERIMENTS AND CALCULATIONS

Having recognized the importance and influence of temperature on the vibrations here as a general sample tests have been performed on a beam and this experiments consists of two parts, that are static and dynamic that each one will be described. But it is worth mentioning the mentioned mechanism the beam can be simulated as a massive spring with stiffness K, which is calculated from equation (1).

\[ K = \frac{3EI}{L^3} \]  

(1)

Where E, is modulus of elasticity, I is the second moment of beam’s area and the beam’s length shown as L.

Another way to calculate the beam’s stiffness, is loading and measuring beam’s free end deflection. The equivalent spring stiffness can be obtained from Equation 2.

\[ K = \frac{F}{\Delta x} \]  

(2)

Where F is the force applied to the spring and Δx is displacement of the spring’s free end.

Here it is not the exact value of the modulus of elasticity so has not been used from theory method for the calculation.

Static experiments

To start the test a belt with approximate dimensions of 110 × 4 × 0.6 cm is welded to the center of the plate, the four bars fixed to the corners for Insulate. Figure 2 shows a schematic of this mechanism. Temperatures that were studied were 60, 80, 100, 120 and 140 ° C, that in each temperature used to the mass of 0.5, 1, 1.5, 2, 3, 3.5 and 4 kg for loading.

The majority of experiments for each temperature performed in two modes - heating and cooling - and three times in each mode, and in the experiments for each mass, the beam’s deflection is measured, and with respect to the more stable temperature conditions during cooling for averaging from the results effect of this mode has been applied twice. In Table 1, the averages of all experiments are presented in static mode.
Table 1 - Average of beam’s stiffness at any temperature.

<table>
<thead>
<tr>
<th>K (N/m)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>873.1</td>
<td>140</td>
</tr>
<tr>
<td>910.46</td>
<td>120</td>
</tr>
<tr>
<td>940.22</td>
<td>100</td>
</tr>
<tr>
<td>982.2</td>
<td>80</td>
</tr>
<tr>
<td>1016.7</td>
<td>60</td>
</tr>
</tbody>
</table>

For better understanding, results are shown in Figure 3. As is clear from the results, it can be stated that the reduction in stiffness with increasing temperature in this range is almost in linear shape. However due to in the high-temperature, changes of modulus of elasticity is not linear, this issue cannot be true for all temperatures. But the range of operating temperatures of mechanical equipment and the structures normally does not exceed these limits, so this linear change will be valid for beam’s stiffness to a great extent. To interpret these results it can be stated, that the increase in temperature causes the expansion of the beam’s volume and then increase the moment of inertia, and on the other hand - according to what is given in the introduction - the modulus of elasticity decreases with increasing temperature. Now, according to equation (1) and that the temperature gradient is relatively low, change in the volume can be ignored, and only effective factor in change of spring stiffness caused by changes in the modulus of elasticity. With what was said, the natural frequency of the system at any step, in free damping state can be obtained, that later it will be used.

Dynamic experiments

Since, in fact there is no ideal spring, and beams also has damping, basically the static experiments use only to reduce unknowns in the equation is the dynamic mode. In this part the general approach to experiments - as already mentioned - is using a vibration meter - that by a sensors that can be installed on beam, the oscillation’s natural frequency will be determined. But due to existence of a vibration meters laboratory device, with little change, it been used. Equipment used universal vibration system that is shown in Figure 4, and consists of a vibrating beam, a motion detector sensor, a damper, a spring and a support to prevent beam falling. The device have a rotary engine with the certain unbalancing, that applied a harmonic force to the system and a sensor that located at the end of beam sign fluctuations and plotted the, amplitude versus time graph. From the device take, spring, damper and hung support apart and then a belt as mentioned in the previous section, the length of 70 cm was welded to the center of the plate and a palate with the same size, and beam fixed to device by nuts and bolts.

When we know that the beam is connected properly to the chassis, A hinge connected to the test beam and another one connected to the device beam - which at that’s end connected to a vibration meters sensor -. Then between these two hinges were welded rod, to transfer force of unbalance motors in the vertical direction and destroy the impact of horizontal force. In Figure 5 the mechanism before the insulation is shown.
Now the existing system can be modeled like Figure 6. Where, ms is mass of spring, Mm is the mass of engine and m is beam’s mass, also in the figure C is structure damping of the beam.

Before expressing system’s oscillation relations, by using the method riley and regardless of the wasting energy by structural damping, spring mass is transferred to the free end of it. Beam deformation due to its own weight can observe in figure 7. The amount of deflection is calculated by equation (3). However due to symbols in the figure other relations can be written.

\[
y = \frac{F}{6EI}(-x^3 + 3Lx^2)
\]  

In the above equation L is length of the beam, y is vertical deflection in the desired length and x is the desired length. To calculate the equivalent mass in the free end of beam, we assume beam’s various particle velocity varies from zero to the value of \(\dot{y}\) from point A to B and we suppose this changes consistent with the static deflection of the beam.

\[
v = \frac{y}{\delta_{max}} \frac{1}{2} L^3 (-x^3 + 3Lx^2)
\]

If will have assume longitudinal density of beam as \(\rho\)

\[
dT = \frac{1}{2} \rho dx \left[ \frac{y}{2L^3}(-x^3 + 3Lx^2) \right]
\]

\[
dT = \frac{1}{2} dm \dot{v}^2
\]

So

\[
T = \frac{\rho y^2}{8L^6} \int (-x^3 + 3Lx^2) \dot{y} dx
\]

\[
T = \frac{\rho y^2}{8L^6} \left[ \frac{x^7}{7} - \frac{9L^2}{5}x^5 - Lx^6 \right]
\]
\[ T = \frac{1}{2} \left( \frac{33\rho L}{140} \right) \theta^3 \]

By replacing \( M_s \) instead of the \( L_p \) equivalent mass will be as follows.
\[ M_{eq} = 0.236 M_s \]  \hspace{1cm} (6)

Now can Expression the equation of motion.
System free-body diagram is shown in Figure 8 and according to that the system frequency characteristics can be obtained.

![System's free-body diagram](image)

Fig 8. System’s free-body diagram

By torque measuring around the point \( o \), system’s equation of motion can be written as follows.
\[
\sum M_o = I \ddot{\theta}
\]
\[ (M_{eq}g l + M_\theta g l_\theta + + mg l_\theta + KL^2) \theta + cl^2 \dot{\theta} + I \ddot{\theta} = 0 \]
\[ M_{eq} g l \theta + M_\theta g l_\theta + + mg l_\theta + + cl^2 \dot{\theta} + KL^2 \theta + I \ddot{\theta} = 0 \]
\[ I = M_{eq} l^2 + M_\theta l_\theta^2 + \frac{1}{3} m l^2 \]
\[ (M_{eq} l^2 + M_\theta l_\theta^2 + \frac{1}{3} m l^2) \theta + (M_{eq} g l^2 + M_\theta g l_\theta^2 + + mg l_\theta^2 + + kl^2) \dot{\theta} + + cl^2 \dot{\theta} = 0 \]  \hspace{1cm} (7)

The above equations for condition that vibrations, not forced, in this case, the natural frequency of undamping and damping system respectively is calculated of equations 8 and 9.

\[ \omega_n = \sqrt{\frac{K}{M_{eq}}} \sqrt{\frac{M_{eq} g l^2 + M_\theta g l_\theta^2 + + mg l_\theta^2 + + KL^2}{M_{eq} l^2 + M_\theta l_\theta^2 + \frac{1}{3} m l^2}} \]  \hspace{1cm} (8)

\[ \omega_d = \omega_n \sqrt{1 - \frac{k^2}{\omega_n^2}} \]  \hspace{1cm} (9)

In equation 8 value of \( K \) for each temperature is obtained from static experiments. The unknowns in the equation 9 are just \( \omega_n \) and \( \xi \). To find \( \omega_n \), unbalance motor operate to applied a certain sinusoidal force is to the system. After starting the motor, we slowly increase motor speed. From device output graphs can be seen that increases motor speed within increasing oscillations amplitude, but after more than a certain limit rates amplitude decreases again, an example of this assertion is shown in Figure 9. As is clear from Figure 9, at a specific speed the amplitude reaches its maximum, That speed represents the natural frequency of damping oscillations \( \omega_n \) and \( \omega_d \) is calculated from equation 10, where \( \tau \) is the period of oscillations in the maximum amplitude and can be obtained from the output graphs.

\[ \omega_d = \frac{2\pi}{\tau} \]  \hspace{1cm} (10)

\[
dT = \frac{1}{2} \rho dx \left[ \frac{y^3}{2L^3} \chi^0 - 3Lx^2 \right] \]
In dynamic experiments – such as static – for the same temperature in tow mode – cooling and heating – output graphs have been recorded and for each temperature the graph that had the greatest amplitude was selected and calculations were performed. For example, the calculation of the natural frequency at 100 °C is shown as follow. And the other temperature results are shown in Table 2. To calculate the natural frequency at 100 °C a graph that seemed a more accurate was chosen that is shown in Figure 10. In this figure the distance between the points of maximum amplitude is 0.256 second. Therefore natural frequency can be calculated as follows:

\[
\omega_n = \frac{2\pi}{T} = \frac{2\pi}{0.256} = 24.54 \text{ rad/s}
\]

From equation 9 and the following values it will have:

L1=L2=50 cm, Mm=3.1 kg, Ms=1.432 kg, m=1.65 kg, L=70 cm, Meq=0.389 kg

Damping ratio can be calculated from equation 9 as follows.

\[
\xi = \sqrt{1 - \left(\frac{\omega_n}{\omega_d}\right)^2} = 0.924
\]

<table>
<thead>
<tr>
<th>T</th>
<th>(\omega_n) (rad/s)</th>
<th>(\omega_d) (rad/s)</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.923</td>
<td>24.54</td>
<td>61.87</td>
<td>140</td>
</tr>
<tr>
<td>.9210</td>
<td>24.54</td>
<td>63.084</td>
<td>120</td>
</tr>
<tr>
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<td>80</td>
</tr>
<tr>
<td>0.929</td>
<td>24.54</td>
<td>66.65</td>
<td>60</td>
</tr>
</tbody>
</table>
Check Results
According to what was at previous sections, can be claimed, according to experiments changes in three important factors were studied:
(A) Equivalent structures stiffness
(B) Structural damping
(C) Natural frequency
Each one will be explained. But before it should be noted at results of high temperatures -120 and 140 °C - , a little heterogeneity can be seen in comparison with results of other temperatures as a result of reduction in temperature uniformity throughout beam.

(A). Equivalent structural stiffness
Stiffness or system resistance against displacement is the most important factor in system’s oscillations that according to the experiments, with increasing temperature, will be decrease. The stiffness decreases with increasing temperature, seems logical, for example, assuming a very high body temperature and assuming it liquid, we can see that stiffness dropped which is consistent with our results. However, another question is how is these changes, according to the results can be said this changes are almost linear. However, it should be noted that these changes may at higher temperatures - and temperatures that structure convert to pasty - changes by nonlinear method. Other notable case that can be expressed in this field is that due to the increased temperature difference between the outside environment and the chamber, heat loss will be increase and actually reach a uniform temperature along the beam at high temperatures is impossible. Therefore it can be stated that with increasing temperature, error at experiments will be increase.

(B). Structural damping
Another important factor affecting on the vibration is systems structural damping, this parameter, were calculated with method that listed in the previous section. But to explain these changes can be said that, with regard to the equation 11, structural damping is dependent on two values, one, C, that is damper coefficient, that can say C is system desire to eliminate vibrations.

\[ \xi = \frac{C}{C_c} \]  
(11)

Another factor in the equation 11, is Cc that can be obtained from the equation 12 and \( \omega_n \) value obtained by using Equation 6.

\[ C_c = \frac{2M_0 \omega_n}{\varphi_2} \]  
(12)

In the previous section was constant with increasing temperature, stiffness decreases, that according to the above equation leads to a reduction Cc , and Cc represents critical damping. According to previous sections and dependence damper coefficient to the physical structure of the system the only way to obtain the amount of structural damping is experiments.

(C). natural frequency
With to the respect previous sections, the natural frequency is dependent on a series of system's physical factors and only due to its high importance in systems vibration, will be study separate, and in fact this is the subject of this experiment and also is a limiting factor in vibrational systems.

Prior to this we mentioned the direct relationship between this parameter and equivalent stiffness, and after applying the structural damping in the calculations, Table 2 was achieved, where was not observed significant changes in the natural frequency with increasing temperature, to explain the stability of the natural frequency in damper mode can be said, according to Equation 9 and Table 2, the amount of damping ratio (\( \xi \)) and undamping natural frequency (\( \omega_n \)), both decrease with increasing temperature and the slope of the decline in the damping ratio is lower and because, always damping ratio is between zero and one, system's natural frequency stabilization - taking into account the damping - with temperature changes in this range appears to be reasonable.

Conclusion
The results of the experiments listed in the following.
As the temperature increases structural damping ratio reduce linearly. This can be seen in Figure 11. As the temperature increases undamping system's natural frequency, will be decrease. In the figure 12 is also visible. As is clear from Figure 13, as the temperature increases, the natural frequency of the damping system almost is constant. Decreasing damping ratio, with increasing temperature is less than reduce the undamping system's natural frequency. This is with regard to the relation 8, it makes natural frequency in damping mode, remain constant. This is shown in Figure 14. Raising the temperature increases the amplitude. An increase in temperature, as shown in the static experiment, the equivalent spring stiffness reduced.
Fig 11. The diagram of damping ratio with increasing temperature

Fig 12. The diagram of undampig system’s natural frequency with increasing temperature

Fig 13. Changing damping system’s natural frequency with changes in temperature

Fig 14. Diagram of comparison gradient of damping ratio and damping system's natural frequency
Suggestions for More Research in This Field

With respect to the fact that even in modeling and analysis software that received temperature conditions as boundary conditions, such as Abaqus software, for each temperature material properties should be identified, so now the only way to analyze the vibration problems at various temperatures is experiment.

Now with respect to clarify the importance of the experiments in this field and that the large and complex systems cannot be examined in this way, one way of measuring the vibrations is using from a vibration meter after construction, as about the bridges is done, but there are other ways to study these systems that seem appropriate, and its use of dimensional analysis and similarity methods. Hence recommended those interested that study about this field. If you need to do experiments that mentioned, is recommended, use from hot gas for heating, and to measure the amplitude and oscillations frequency use from, vibration meter portable sensors that can be installed on parts and can be connected to computer.

References

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